

# Generalized Dirichlet normal ordering in open bosonic strings

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## Abstract

Generally, open string boundary conditions play a nontrivial role in string theory. For example, in the presence of an antisymmetric tensor background field, they will lead the spacetime coordinates noncommutative. In this paper, we mainly discuss how to build up a generalized Dirichlet normal ordered product of open bosonic string embedding operators that satisfies both the equations of motion and the generalized Dirichlet boundary conditions at the quantum level in the presence of an antisymmetric background field, as the generalized Neumann case has already been discussed in the literature. Further, we also give a brief check of the consistency of the theory under the newly introduced normal ordering.

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## I. INTRODUCTION

Recent developments in string theory [1, 2, 3] suggest scenarios that our four-dimensional spacetime with the standard model fields corresponds to a D3-brane [4] embedded in a larger manifold, on which the open string endpoints attach. One important consequence of such models is that, in the presence of an antisymmetric tensor background field, the open string boundary conditions lead to the noncommutativity of the spacetime coordinates on the branes. This ‘noncommutative geometry’ now has proven to play a key role in the dynamics of D-branes, and is regarded to provide an important hint about the nature of the spacetime at very small length scales [5, 6, 7, 8], which is one of the reasons for the increasing interests in studying noncommutative quantum field theories [8, 9]. Furthermore, this also illustrates the fact that the open string boundary conditions may play a nontrivial role in string theory and in our four-dimensional physics.

In the context of the quantum field theory, to preserve causality, products of quantum fields must be defined at the same spacetime points. But to do this, one also introduces divergences. The same thing holds true in string theory, for example, products of the string embedding operators on the same world-sheet points are in general singular objects. This situation is well known and one can remove the singular parts of the operator products by defining proper normal ordered objects. Usually in string theory, the normal ordered products of the conformal operators on the string world-sheet are defined so as to satisfy the classical equations of motion at the quantum level. Also for the open string case, as the world-sheet has boundaries, they have to satisfy corresponding boundary conditions. This has already been studied by Braga *etc.* In a recent paper [11], they defined a new normal ordered product for open string embedding operators which satisfy both the equations of motion and the boundary conditions with a constant antisymmetric tensor background field present. But what they considered was actually only the generalized Neumann case in the world-volume of the D-branes, whose boundary conditions are a mixture of the simple Neumann and Dirichlet ones. Meanwhile the two boundary conditions can also mix in another way to form so-called generalized Dirichlet boundary conditions in the directions perpendicular to the D-branes which represent new information. So it is quite necessary to study this generalized Dirichlet case.

In this paper, after a brief review of the newly defined Neumann normal ordering in Ref.

[11] in Sec. II, we will give a detailed discussion on a normal ordered operator product which satisfies both the equations of motion and the generalized Dirichlet boundary conditions in Sec. III. As the presence of the antisymmetric background field, the discussion similar to that of the Neumann case will not be explicit, and so we take another approach, namely we consider the  $d$ -dimensional spacetime with some periodic compactified dimensions, and take the radius of these dimensions  $R \rightarrow 0$ , which leads to corresponding  $T$ -dual noncompact dimensions, leaving several D-branes at some places. Then we will obtain the new Dirichlet boundary conditions and define a new generalized Dirichlet normal ordered operator product in the  $T$ -dual picture of the theory. We also check some further issues in Sec. IV, such as the central charge gets no impact, and the spacetime coordinates of the string endpoints in the directions parallel and perpendicular to the D-branes both become noncommutative under these generalized Dirichlet and Neumann normal orderings. Finally, the conclusion is given in Sec. V.

## II. REVIEW OF NEUMANN NORMAL ORDERING

The generalized normal ordered operator product for the open bosonic string which satisfies both the equations of motion and the generalized Neumann boundary conditions in the D-brane world-volume has recently been discussed in Ref. [11] and further in Ref. [12]. Here we give them a little detailed review, as in which many results will be useful in the next section. We generally use the same symbols as in Ref. [10] in all this paper.

The classical action for a bosonic string in the presence of a constant antisymmetric tensor background field  $B_{\mu\nu}$  is

$$S = \frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma (g^{ab}\eta_{\mu\nu} + i\epsilon^{ab}B_{\mu\nu})\partial_a X^\mu \partial_b X^\nu, \quad (1)$$

where  $X^\mu$  are the spacetime embedding coordinates of the bosonic string,  $\Sigma$  is the string world-sheet parameterized by  $\sigma^1 = \sigma, \sigma^2 = i\tau$  with the boundary (string endpoints) for open string at  $\sigma = 0, \pi$ ,  $g_{ab}$  is the Euclidean world-sheet metric with signature  $(+, +)$ ,  $\epsilon^{ab}$  is the antisymmetric tensor with  $\epsilon^{12} = 1$ , and  $\eta_{\mu\nu} = \text{diag}(-, +, \dots, +)$  is the flat Minkowski spacetime metric.

The variation of the action (1) with respect to  $X^\mu$  gives the equations of motion

$$(\partial_1^2 + \partial_2^2)X^\mu = 0, \quad (2)$$

and the boundary conditions

$$(\eta_{\mu\nu}\partial_1 + iB_{\mu\nu}\partial_2)X^\nu|_{\sigma=0,\pi} = 0. \quad (3)$$

Comparing to the original Neumann boundary conditions  $\partial_1 X^\mu|_{\sigma=0,\pi} = 0$  [10] obtained with  $B_{\mu\nu} = 0$ , Eqs. (3) mix both the Neumann and Dirichlet boundary conditions, but we will simply call them the generalized Neumann boundary conditions, to be consistent with the discussion in the next section.

It is convenient to express the above equations in terms of complex world-sheet coordinates. Define  $w = \sigma^1 + i\sigma^2$ ,  $\bar{w} = \sigma^1 - i\sigma^2$ , and further  $z = -e^{-iw}$ ,  $\bar{z} = -e^{i\bar{w}}$  [10]. As discussed in Ref. [11], the classical action (1) then takes the form

$$S = \frac{1}{2\pi\alpha'} \int_{\Sigma} d^2z (\eta_{\mu\nu} + B_{\mu\nu}) \partial_z X^\mu \partial_{\bar{z}} X^\nu, \quad (4)$$

and the classical equations of motion (2) and the generalized Neumann boundary conditions (3) take the forms

$$\partial_z \partial_{\bar{z}} X^\mu(z, \bar{z}) = 0, \quad (5)$$

$$[\eta_{\mu\nu}(\partial_z - \partial_{\bar{z}}) - B_{\mu\nu}(\partial_z + \partial_{\bar{z}})] X^\nu|_{z=\bar{z}} = 0. \quad (6)$$

One can also study the properties of quantum operators by considering the expectation values of the corresponding classical objects. Define the value of an operator  $\mathcal{F}$  by the path integral

$$\langle \mathcal{F}[X] \rangle = \int [dX] \exp(-S) \mathcal{F}[X]. \quad (7)$$

As discussed in Ref. [10], by using the fact that the path integral of a total derivative is zero,

$$0 = \int [dX] \frac{\delta}{\delta X^\mu(z, \bar{z})} \exp(-S[X]) \quad (8)$$

will suggest that it holds the quantum versions of the equations of motion (5) and the generalized Neumann boundary conditions (6) (for more details, to see Ref. [11])

$$\partial_z \partial_{\bar{z}} \hat{X}^\mu(z, \bar{z}) = 0, \quad (9)$$

$$[\eta_{\mu\nu}(\partial_z - \partial_{\bar{z}}) - B_{\mu\nu}(\partial_z + \partial_{\bar{z}})] \hat{X}^\nu|_{z=\bar{z}} = 0, \quad (10)$$

where  $\hat{X}^\mu$  now are the embedding operators of the string, which means that these equations hold at the quantum level. Further, products of operators at the same world-sheet points

yield a singular behavior. By calculating

$$0 = \int [dX] \frac{\delta}{\delta X^\mu(z, \bar{z})} [\exp(-S[X]) X^\nu(z', \bar{z}')], \quad (11)$$

one finds the corresponding products of operators satisfy

$$\frac{1}{\pi\alpha'} \partial_z \partial_{\bar{z}} \hat{X}^\mu(z, \bar{z}) \hat{X}^\nu(z', \bar{z}') = -\eta^{\mu\nu} \delta^2(z - z', \bar{z} - \bar{z}'), \quad (12)$$

$$[\eta_{\mu\nu}(\partial_z - \partial_{\bar{z}}) - B_{\mu\nu}(\partial_z + \partial_{\bar{z}})] \hat{X}^\nu(z, \bar{z}) \hat{X}^\rho(z', \bar{z}')|_{z=\bar{z}} = 0. \quad (13)$$

If defining a normal ordered product of two embedding operators in the standard way [10]

$$: \hat{X}^\mu(z, \bar{z}) \hat{X}^\nu(z', \bar{z}') := \hat{X}^\mu(z, \bar{z}) \hat{X}^\nu(z', \bar{z}') + \frac{\alpha'}{2} \eta^{\mu\nu} \ln |z - z'|^2, \quad (14)$$

it will satisfy the equations of motion (12) but fail to satisfy the generalized Neumann boundary conditions (13) at the quantum level. To reconcile this problem, Braga *etc* recently introduced a new normal ordering [11]

$$\begin{aligned} : \hat{X}^\mu(z, \bar{z}) \hat{X}^\nu(z', \bar{z}') : &= \hat{X}^\mu(z, \bar{z}) \hat{X}^\nu(z', \bar{z}') + \frac{\alpha'}{2} \eta^{\mu\nu} \ln |z - z'|^2 \\ &+ \frac{\alpha'}{2} ([\eta - B]^{-1} [\eta + B])^{\mu\nu} \ln(z - \bar{z}') + \frac{\alpha'}{2} ([\eta - B] [\eta + B]^{-1})^{\mu\nu} \ln(\bar{z} - z'). \end{aligned} \quad (15)$$

It is easy to check that this normal ordered operator product satisfies both the equations of motion (12) and the generalized Neumann boundary conditions (13) at the quantum level. To be consistent with the following discussion, here we name this new normal ordered product as the second Neumann normal ordering of the open bosonic string. When taking the antisymmetric background field  $B_{\mu\nu} = 0$ , it reduces to the familiar form

$$: \hat{X}^\mu(z, \bar{z}) \hat{X}^\nu(z', \bar{z}') := \hat{X}^\mu(z, \bar{z}) \hat{X}^\nu(z', \bar{z}') + \frac{\alpha'}{2} \eta^{\mu\nu} \ln |z - z'|^2 + \frac{\alpha'}{2} \eta^{\mu\nu} \ln |z - \bar{z}'|^2, \quad (16)$$

which is correspondingly called the first Neumann normal ordering.

### III. DIRICHLET NORMAL ORDERING

To obtain the above generalized Neumann normal ordering, the authors implicitly only considered things in the D-brane world-volume and used the assumptions that the field  $B = B_{\mu\nu} dX^\mu \wedge dX^\nu$  and further the boundary conditions  $X^\alpha|_{\sigma=0,\pi} = x_0^\alpha$  (namely  $\delta X^\alpha|_{\sigma=0,\pi} = 0$ ) [5, 6, 13], where now we use the indices  $\mu, \nu$  to denote the directions along which the D-branes are expanded and the indices  $\alpha, \beta$  to denote the directions that are perpendicular

to the D-branes. Here we note that generally it is not quite proper to use the simple boundary conditions  $X^\alpha|_{\sigma=0,\pi} = x_0^\alpha$ , as they only allow flat and static D-branes, but which can actually be dynamical, especially with nontrivial antisymmetric field components  $B_{\alpha\beta}$ . So we need to generalize them in the directions perpendicular to the D-branes. Also to make the discussion simple and physically clear, we consider the case for the background field  $B = B_{\mu\nu}dX^\mu \wedge dX^\nu + B_{\alpha\beta}dX^\alpha \wedge dX^\beta$ .

Recall from the toroidal compactification and  $T$ -duality theories [10] that when a space-time dimension of a string theory is periodically compactified  $X \cong X + 2\pi R$ , the field  $X$  on the world-sheet will split into holomorphic and antiholomorphic parts

$$X(z, \bar{z}) = X_L(z) + X_R(\bar{z}). \quad (17)$$

Then if taking the radius  $R \rightarrow 0$ , it will lead to a new theory with a noncompact dimension described by

$$X'(z, \bar{z}) = X_L(z) - X_R(\bar{z}), \quad (18)$$

with several D-branes leaving at some places, on which the open string endpoints are fixed. Generally, these are the same theory, one written in terms of  $X$  and one in terms of  $X'$ . And the equivalence is known as  $T$ -duality. One of the most important properties of  $T$ -duality is that it interchanges the original Neumann and Dirichlet boundary conditions,

$$\partial_1 X = -i\partial_2 X', \quad \partial_2 X = i\partial_1 X'. \quad (19)$$

Now consider some originally compactified dimensions  $X^\alpha$  in the whole spacetime. The equations of motion and the generalized Neumann boundary conditions with nontrivial  $B_{\alpha\beta}$  present are still expressed by (2) and (3) (only by changing the indices  $\mu, \nu$  to  $\alpha, \beta$ ). But when taking  $R \rightarrow 0$ , in the  $T$ -dual picture, by considering (19), they generally change to the forms

$$(\partial_1^2 + \partial_2^2)X'^\alpha = 0, \quad (20)$$

$$(\eta_{\alpha\beta}\partial_2 - iB_{\alpha\beta}\partial_1)X'^\beta|_{\sigma=0,\pi} = 0. \quad (21)$$

From these equations, we see that the equations of motion do not change the forms, but the forms of the boundary conditions change. Similar to the generalized Neumann boundary conditions (3), these new ones can be called the generalized Dirichlet boundary conditions,

though they are also a mixture of the original Neumann and Dirichlet ones. In these equations we use the prime to denote the  $T$ -dual dimensions. But since all the following discussions will be carried out in the  $T$ -dual noncompact dimensions, we will drop the prime for simplicity. And here we can conclude the general spacetime picture we will use in following and actually implicitly used in the above section: we generally take  $d$ -dimensional noncompact spacetime  $X^M (M = 0, \dots, d-1)$ , with several  $Dp$ -branes expanded in some directions  $\alpha = p+1, \dots, d-1$ . (Also we should specify that all the calculations in the above section were done in the world-volume of the D-branes only).

To express the above equations (22) and (23) in terms of complex world-sheet coordinates, it gives that

$$\partial_z \partial_{\bar{z}} X^\alpha = 0, \quad (22)$$

$$[\eta_{\alpha\beta}(\partial_z + \partial_{\bar{z}}) - B_{\alpha\beta}(\partial_z - \partial_{\bar{z}})] X^\beta|_{z=\bar{z}} = 0. \quad (23)$$

A similar path integral method show that these equations of motion (22) and generalized Dirichlet boundary conditions (23) also have quantum versions, namely the corresponding operators satisfy

$$\partial_z \partial_{\bar{z}} \hat{X}^\alpha(z, \bar{z}) = 0, \quad (24)$$

$$[\eta_{\alpha\beta}(\partial_z + \partial_{\bar{z}}) - B_{\alpha\beta}(\partial_z - \partial_{\bar{z}})] \hat{X}^\beta|_{z=\bar{z}} = 0, \quad (25)$$

at the quantum level. Further, the products of operators at the same world-sheet points also yield a singular behavior,

$$\frac{1}{\pi\alpha'} \partial_z \partial_{\bar{z}} \hat{X}^\alpha(z, \bar{z}) \hat{X}^\beta(z', \bar{z}') = -\eta^{\alpha\beta} \delta^2(z - z', \bar{z} - \bar{z}'), \quad (26)$$

$$[\eta_{\alpha\beta}(\partial_z + \partial_{\bar{z}}) - B_{\alpha\beta}(\partial_z - \partial_{\bar{z}})] \hat{X}^\beta(z, \bar{z}) \hat{X}^\rho(z', \bar{z}')|_{z=\bar{z}} = 0. \quad (27)$$

Again the standard normal ordering (14) satisfies the equations of motion (26) but fails to satisfy the generalized Dirichlet boundary conditions (27) at the quantum level. And to reconcile this problem, similar to the second Neumann normal ordering (15), we define a new normal ordering as

$$\begin{aligned} : \hat{X}^\alpha(z, \bar{z}) \hat{X}^\beta(z', \bar{z}') : &= \hat{X}^\alpha(z, \bar{z}) \hat{X}^\beta(z', \bar{z}') + \frac{\alpha'}{2} \eta^{\alpha\beta} \ln |z - z'|^2 \\ &\quad - \frac{\alpha'}{2} ([\eta - B]^{-1} [\eta + B])^{\alpha\beta} \ln(z - z') - \frac{\alpha'}{2} ([\eta - B] [\eta + B]^{-1})^{\alpha\beta} \ln(\bar{z} - \bar{z}'), \end{aligned} \quad (28)$$

which satisfies both the equations of motion (26) and the generalized Dirichlet boundary conditions (27) at the quantum level. Comparing to the second Neumann normal ordering (15), this new one should be named as the second Dirichlet normal ordering of the open bosonic string. When taking the antisymmetric background field  $B_{\alpha\beta} = 0$ , it reduces to another familiar form

$$: \hat{X}^\alpha(z, \bar{z}) \hat{X}^\beta(z', \bar{z}') := \hat{X}^\alpha(z, \bar{z}) \hat{X}^\beta(z', \bar{z}') + \frac{\alpha'}{2} \eta^{\alpha\beta} \ln |z - z'|^2 - \frac{\alpha'}{2} \eta^{\alpha\beta} \ln |z - \bar{z}'|^2, \quad (29)$$

which is correspondingly called the first Dirichlet normal ordering.

It is interesting to notice that the above second Neumann (15) and Dirichlet (28) normal orderings are quite likely, only with the signatures of the additional terms being opposite. These additional terms can be understood as an ‘image’ charge contribution as in electrostatics, but with different signs. For the second Neumann normal ordering (15), the image charge takes the same signature as the original charge, and for the second Dirichlet normal ordering (28), the two charges take the opposite signatures. This appears especially clear for the antisymmetric field  $B = 0$  case (16) and (29), which is already known. Further comparing Eqs. (15) and (28) to Eqs. (16) and (29), it also suggests a physical manifestation of the antisymmetric background field  $B$  here: it reflects the affection of one direction by the other ones. In the case discussed here, as we have supposed  $B_{\mu\alpha} = 0$ , a direction perpendicular to the D-branes (or to say, an originally compactified dimension) can only be affected by other perpendicular directions through the second Dirichlet normal ordering, and the same thing holds in the D-brane world-volumes (or to say, the originally noncompact dimensions) through the second Neumann normal ordering. In the more general case with nontrivial  $B_{\mu\alpha}$  components, the two different sets of directions can also affect each other through the two generalized normal orderings.

#### IV. FURTHER ISSUES

Now that we have obtained the generalized Neumann and Dirichlet normal orderings of the open bosonic string in the presence of a constant antisymmetric background field, for



any arbitrary functional operators  $\mathcal{F}[X]$  and  $\mathcal{G}[X]$ , their OPE is generalized to

$$\begin{aligned} : \mathcal{F} :: \mathcal{G} : &= \exp \left( -\frac{\alpha'}{2} \int d^2 z_1 d^2 z_2 [\eta^{\mu\nu} \ln |z_1 - z_2|^2 + ([\eta - B]^{-1} [\eta + B])^{\mu\nu} \ln(z_1 - \bar{z}_2) \right. \\ &\quad \left. + ([\eta - B][\eta + B]^{-1})^{\mu\nu} \ln(z_1 - \bar{z}_2)] \frac{\delta}{\delta X_F^\mu(z_1, \bar{z}_1)} \frac{\delta}{\delta X_G^\nu(z_2, \bar{z}_2)} \right) : \mathcal{F} \mathcal{G} : \end{aligned} \quad (30)$$

in the directions along which the D-branes expand by using the second Neumann normal ordering (15), and to

$$\begin{aligned} : \mathcal{F} :: \mathcal{G} : &= \exp \left( -\frac{\alpha'}{2} \int d^2 z_1 d^2 z_2 [\eta^{\alpha\beta} \ln |z_1 - z_2|^2 - ([\eta - B]^{-1} [\eta + B])^{\alpha\beta} \ln(z_1 - \bar{z}_2) \right. \\ &\quad \left. - ([\eta - B][\eta + B]^{-1})^{\alpha\beta} \ln(z_1 - \bar{z}_2)] \frac{\delta}{\delta X_F^\alpha(z_1, \bar{z}_1)} \frac{\delta}{\delta X_G^\beta(z_2, \bar{z}_2)} \right) : \mathcal{F} \mathcal{G} : \end{aligned} \quad (31)$$

in the directions perpendicular to the D-branes by using the second Dirichlet normal ordering (28), where the functional derivatives act only on the fields in  $\mathcal{F}$  or  $\mathcal{G}$  respectively. And their complete OPE form is the sum of these two forms.

Then it is important to check whether the theory is still consistent under these changes. The first issue is the central charge, which is a purely quantum effect and takes a central role in deciding the critical dimensions of the string theory. Actually the check is quite simple. The world-sheet energy-momentum tensor is

$$T(z) = -\frac{1}{\alpha'} : \partial_z X^M(z) \partial_z X_M(z) :, \quad (32)$$

where  $M$  sums over  $0, \dots, d-1$ , and we neglect the hats on the embedding operators. Then one sees that as  $\partial_z \bar{z} = \partial_{\bar{z}} z = 0$ , it gives

$$\partial_z \ln(\bar{z} - z') = \partial_{\bar{z}} \ln(z - \bar{z}') = \partial_{z'} \ln(z - \bar{z}') = \partial_{\bar{z}'} \ln(\bar{z} - z') = 0. \quad (33)$$

In the directions parallel to the D-branes, by using the second Neumann normal ordering (15), a direct calculation yields

$$\begin{aligned} : \partial_z X^\mu(z) \partial_z X_\mu(z) :: \partial_{z'} X^\nu(z') \partial_{z'} X_\nu(z') : &\sim \frac{(p+1)\alpha'^2}{2(z-z')^4} - \frac{2\alpha'}{(z-z')^2} : \partial_{z'} X^\mu(z') \partial_{z'} X_\mu(z') : \\ &\quad - \frac{2\alpha'}{z-z'} : \partial_{z'}^2 X^\mu(z') \partial_{z'} X_\mu(z') :, \end{aligned} \quad (34)$$

and in the directions perpendicular to the D-branes, by using the second Dirichlet normal ordering (28),

$$\begin{aligned} : \partial_z X^\alpha(z) \partial_z X_\alpha(z) :: \partial_{z'} X^\beta(z') \partial_{z'} X_\beta(z') : &\sim \frac{(d-1-p)\alpha'^2}{2(z-z')^4} - \frac{2\alpha'}{(z-z')^2} : \partial_{z'} X^\alpha(z') \partial_{z'} X_\alpha(z') : \\ &\quad - \frac{2\alpha'}{z-z'} : \partial_{z'}^2 X^\alpha(z') \partial_{z'} X_\alpha(z') :, \end{aligned} \quad (35)$$

where  $\sim$  means ‘equal up to nonsingular terms’, and  $p$  counts the number of spatial dimensions of the D-branes, which suggests that in (34) the indices  $\mu, \nu$  sum over only the dimensions parallel to the D-branes and in (35)  $\alpha, \beta$  over the dimensions perpendicular to the D-branes. Combining these two relations implies that

$$T(z)T(z') \sim \frac{d}{2(z-z')^4} + \frac{2}{(z-z')^2}T(z') + \frac{1}{z-z'}\partial_{z'}T(z'), \quad (36)$$

which is the same as the standard  $TT$  OPE obtained by using the original normal ordering (14) with the antisymmetric field  $B = 0$ . Here it should be noted that only by considering both the second Neumann and Dirichlet normal orderings in the two respect sets of directions can one get a complete and correct result (and so it is not quite correct as did in Ref. [12], for they only considered the case in the D-brane world-volume). The same relation holds for the  $\tilde{T}\tilde{T}$  OPE. Therefore, we get the results that the Virasoro algebra remains unchanged and the two new generalized normal orderings have no impact on the central charge.

Next we check the normal ordered commutators of the embedding operators of the open bosonic string. For the second Neumann normal ordering (15), it was discussed in Ref. [11], with the result in terms of the  $\sigma^1, \sigma^2$  coordinates that

$$: [X^\mu(\sigma^1, \sigma^2), X^\nu(\sigma^1, \sigma'^2)] := [X^\mu(\sigma^1, \sigma^2), X^\nu(\sigma^1, \sigma'^2)], \quad (37)$$

and for the second Dirichlet normal ordering (28), a totally similar calculation shows that the result (37) still holds (only by changing  $\mu, \nu$  to  $\alpha, \beta$ ), which together suggest that the commutators do not get any extra contributions from these two generalized normal ordering prescriptions. Then the equal time commutators, expressed by identifying  $\tau = \tau'$ , i.e.  $\sigma^2 = \sigma'^2$  and setting  $\sigma^1 = \sigma, \sigma'^1 = \sigma'$ , are given

$$[X^\mu(\tau, \sigma), X^\nu(\tau, \sigma')] = 2i\alpha'(M^{-1}B)^{\mu\nu} \left[ \sigma + \sigma' - \pi + \sum_{m \neq 0} \frac{1}{m} \sin m(\sigma + \sigma') \right], \quad (38)$$

in the D-brane world-volume [12] and further a similar calculation suggests

$$[X^\alpha(\tau, \sigma), X^\beta(\tau, \sigma')] = -2i\alpha'(M^{-1}B)^{\alpha\beta} \left[ \sigma + \sigma' - \pi + \sum_{m \neq 0} \frac{1}{m} \sin m(\sigma + \sigma') \right]. \quad (39)$$

in the directions perpendicular to the D-branes. As the infinite series give

$$\sum_{m \neq 0} \frac{1}{m} \sin m(\sigma + \sigma') = \pi - (\sigma + \sigma')$$

for  $\sigma, \sigma' \in (0, \pi)$  and on the boundaries of the string world-sheet

$$\sum_{m \neq 0} \frac{1}{m} \sin m(\sigma + \sigma')|_{\sigma, \sigma' = 0, \pi} = 0,$$

these commutation relations show that the coordinates of string endpoints in the directions both parallel and perpendicular to the D-branes become noncommutative, while in the internal of the open string world-sheet, they are still commutative.

One can check other issues, such as the mode expansions of the bosonic string [12], and generally one finds that though the spectrum is shifted in the presence of the antisymmetric field, the theory is still consistent and the world-sheet is still a CFT, a good string background.

## V. CONCLUSION

Generally the boundary conditions for the open string take an important role in many aspects of the string theory, such as the quantization of the open string, the noncommutativity of the spacetime embedding coordinates, etc. So they have been detailed discussed in the literature, especially with a newly defined normal ordered operator product, in the presence of a constant antisymmetric background field. But meanwhile most discussions only focused on the generalized Neumann case in the D-brane world-volume. And so in this paper, after giving a brief review of the Neumann case, we discussed the generalized Dirichlet case in the directions perpendicular to the D-branes, by using a quite technical approach that we take the radius of some periodic compactified dimensions to zero and obtain the generalized Dirichlet boundary conditions in the  $T$ -dual picture. Then to satisfy both the equations of motion and the generalized Dirichlet boundary conditions, we defined a new normal ordered product of the embedding operators. Finally we discussed that both the generalized Neumann and Dirichlet normal orderings, which are quite likely, though lead the spacetime coordinates of the string endpoints in the directions both parallel and perpendicular to the D-branes noncommutative, but have no impact on the central charge, and retain the consistency of the theory.

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